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Seat No. _______________

HAJ-003-1015001

B. Sc. (Sem. V) (CBCS) Examination

May - 2023

Mathematics : Paper - M - 05 (A)

(Mathematical Analysis - 1 & Abstract Algebra - 1)

(New Course)

Faculty Code : 003

Subject Code : 1015001

Time : $2\frac{1}{2}$ / Total Marks : **70**

1 (A) Answer the following questions in short : **4**

(1) Define Neighborhood

(2) Give an example of a subset of metric space R which is not open and closed

(3) Define Interior point

(4) Define Dense Set

(B) Attempt any one out of two : **2**

(1) Obtain border set of the subset (1, 3) of metric space *R*.

(2) Determine whether set $\{x \in R / x^2 - 4x + 4 = 0\}$ is open or closed set.

(C) Attempt any one out of two : **3**

(1) State and prove principle of Housedorff's in metric space.

(2) Prove that the finite intersection of open sets of metric space is an open set.

(D) Attempt any one out of two : **5**

(1) Prove that Closer Set of any subset of a metric space is a closed set.

(2) Let (X, d) be a metric space and $a \in X$ then prove that $N(a, \delta)$ is an open set.

2 (A) Answer the following questions in short : **4**

(1) Define Upper Riemann sum.

(2) Define Finer partition.

(3) Define Riemann Integration.

(4) Define norm of a partition.

(B) Attempt any one out of two : **2**

(1) If $f : [0, 1] \to R, f(x) = x$ and $P = \{0, 1/2, 1\}$ then find $U(P, f)$

(2) Prove that every constant function is Riemann integrable

(C) Attempt any one out of two : **3**

(1) If $f(x) = [x]$, $x \in [0, 3]$ then show that $f \in R_{[0, 3]}$ and find $\int_0^3 f(x)\,dx$, where $[x]$ denote the greatest integer not greater than x.

(2) If f is continuous on $[a, b]$ then prove that f is Riemann Integral on $[a, b]$.

(D) Attempt any one out of two : **5**

(1) If f is monotonic on $[a, b]$ then prove that f is Riemann Integral on $[a, b]$.

(2) State and prove necessary and sufficient condition for a bounded function f defined on $[a, b]$ to be R-integrable

3 (A) Answer the following questions in short : **4**

(1) Define Integral function.

(2) Define abelian group.

(3) Define special linear group.

(4) Define group.

(B) Attempt any one out of two : **2**

(1) Convert $\lim_{n\to\infty} \frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2}$ as definite integral.

(2) Prove that identity element in a group is unique.

(C) Attempt any one out of two **3**

(1) State and prove First mean value theorem of integral calculus.

(2) If $(G,*)$ is a group then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.

(D) Attempt any one out of two : **5**

(1) Prove that $\frac{\pi^3}{51} \leq \int_0^{\pi} \frac{x^2}{10+7\cos x} dx \leq \frac{\pi^3}{9}$.

(2) Show that $(Z_n, +_n)$ is a group, where $n \in N$.

4 (A) Answer the following questions in short : **4**

(1) Define symmetric group.

(2) Define cyclic subgroup.

(3) Define centre of a group.

(4) Define permutation.

(B) Attempt any one out of two : **2**

(1) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}, \sigma \in S_5$, then find σ^{-1}.

(2) Check that permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 1 & 5 & 6 \end{pmatrix}$ is even or odd

(C) Attempt any one out of two **3**

(1) Let $H \leq G$ and $a, b \in G$ then prove that $He = H$ and $a \in Ha$.

(2) If G is a group, then prove that $o(a) / o(G)$; $\forall\ a \in G$.

(D) Attempt any one out of two : **5**

(1) State and prove Lagrange's theorem.

(2) Prove that the set A_n of all even permutations of S_n ($n \geq 2$) is a subgroup of S_n of order $\frac{n!}{2}$.

5 (A) Answer the following questions in short : **4**

(1) Define Inner Automorphism

(2) Define Normal subgroup

(3) Define simple group.

(4) Define Isomorphism

(B) Attempt any one out of two : **2**

(1) If a finite group G has only one subgroup H of given order, then prove H is a Normal subgroup of G.

(2) Let G be a group and let $H = \left\{a^2 / a \in G\right\} \leq G$. Then show that H is a normal Subgroup of G.

(C) Attempt any one out of two : **3**

(1) Prove that a subgroup of index 2 in a group is a normal subgroup.

(2) A subgroup H of a group G is a normal subgroup of $G \Leftrightarrow aHa^{-1} \subset H; \forall \alpha \in G$.

(D) Attempt any one out of two : **5**

(1) State and prove Cayle's theorem.

(2) Show that $(R_+, \bullet) \cong (R,+)$.
